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ABSTRACT

Various researchers have associated meaningful problem solving with methods guided directly by a conceptual knowledge base. By contast, a meaningless solving course, or sequence of operations, is essentially independent of the solver's conceptual understanding of the problem under consideration. This paper is the first to document a meaningless, and nonalgorithmic, student solving course that generated a correct solution to an unfamiliar problem within the context of a secondary biology class. The study originally was designed to constrain the meaningful solutions of 14 outstanding science students to a graphing problem about osmosis. Each solver generated a think-aloud solution to the problem, a pencil-and-paper solution, and a retrospective report of the solution process. Data were utilized to assess whether the solving course was meaningful, the solution was correct, and the student indeed had no prior knowledge of the problem and/or solution. Thirteen of the 14 solutions were meaningful. The one meaningless solving course provides the grist for the discussion and demonstrates that: (1) an unfamiliar problem need not be solved meaningfully to be solved correctly; and (2) a meaningless solution process need not be characteristically algorithmic. (38 references) (JJK)

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A Meaningless but Nonalgorithmic Solving Course: Solving a Graphing Problem About Osmosis by Analogy

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ABSTRACT

A meaningless solving course is essentially independent of the solver's conceptual knowledge. This paper is the first to document a meaningless but nonalgorithmic solving course that generated a correct solution to an unfamiliar problem. This solving course is significant for two reasons. It demonstrates that (a) an unfamiliar problem need not be solved meaningfully to be solved correctly and (b) a meaningless solving course need not be algorithmic. The study was designed to constrain the meaningful solving of a graphing problem about osmosis. The solvers were 14 outstanding high school science students. They were also unfamiliar with the problem. Each solver generated a think-aloud solving of the problem, a pencil-and-paper solution, and a retrospective report of the solving course. The data were used to assess whether the solving course was meaningful, the solution was correct, and the solver had been unfamiliar with the problem. Thirteen of the 14 solving courses were meaningful. The one meaningless solving course was an example of the solving of a problem by analogy.

A MEANINGLESS BUT NONALGORITHMIC SOLVING COURSE: SOLVING A GRAPHING PROBLEM ABOUT OSMOSIS BY ANALOGY

Introduction

Meaningful and meaningless problem solving

Various researchers associate problem solving with at least two distinct kinds of personal knowledge: conceptual and procedural (Anderson, 1983; Gagne, 1968; Greeno, 1973; Larkin, 1980; McMillan & Swadener, 1991; Ryle, 1949; Shuell, 1985; Siegler, 1986; Stewart, 1982b). Conceptual knowledge is knowledge of the concepts, laws, and theories of a domain (Stewart, 1982b). Procedural knowledge is knowledge of how to execute a sequence of operations. Conceptual knowledge provides meaning for the procedures (Stewart, 1982b). Problem solving that is guided primarily by conceptual knowledge is known, therefore, as meaningful problem solving (Stewart, 1982a).

For the study described here, an attempt was made to constrain the meaningful solving of a graphing problem about osmosis. Accordingly, most of the solvers constructed a solving course that was guided primarily by conceptual knowledge. Solving course refers here to a sequence of solving operations. The purpose of this paper is to report on the one solving course that was guided primarily by procedural knowledge.

Procedural knowledge consists of heuristics and algorithms. Heuristics are broadly applicable, content-independent, solving strategies (Polya, 1957). Polya identified solving a problem by analogy as a kind of heuristic. An analogue with a known solving course serves as the model for the solving of a problem. The solving course discussed in the paper undertaken here is an example of the solving of a problem by analogy.

In contrast to a heuristic, an algorithm is a precise routine that has been specified to solve problems of a particular class. An algorithm is necessarily symbolic. Symbols facilitate the application of an algorithm to the various problems within the class (Landa, 1974). The symbols, however, obliterate the meaning of the operations and thus enable the solving course to be executed meaninglessly (Landa, 1974). A meaningless solving course is essentially independent of the solver's conceptual knowledge.

For this study, in order to constrain meaningful solving courses, it was stipulated that each solver be unfamiliar with the problem. This is not to say that the kind of problem must be absolutely new. In fact, an absolutely new kind of problem cannot be solved (Polya, 1957). Rather solvers must be unfamiliar with some aspect of the problem statement, such as the problem condition. Various researchers have affirmed

the connection between the solving of an unfamiliar problem and the solver's conceptual knowledge (Greeno, 1978; Novak, 1977; Novak, Govin, & Johansen, 1983; Wertheimer, 1959). Solvers unfamiliar with a problem are unlikely to execute a prescribed algorithm. Although access to prescribed algorithms was interdicted in this study, one solving course was, nevertheless, meaningless.

The paper undertaken here is the first to document a meaningless but nonalgorithmic solving course wherein a correct solution was generated to an unfamiliar problem. This solving course is significant for two reasons: It has been assumed that an unfamiliar problem must be solved meaningfully to be solved correctly (Novak, 1977; Wertheimer, 1959). Secondly, this solving course demonstrates that procedures other than algorithms can generate correct solutions meaninglessly. Heretofore, the meaningless solving courses identified in the science education literature that have generated a correct solution have been the execution of a prescribed algorithm (Gabel, Sherwood, & Enochs, 1984; Lythcott, 1990; Stewart, 1983; Yarroch, 1985).

The Research Event

The subjects for this interpretive study were high school students who had been unfamiliar with a particular problem about osmosis. In as much as most

students are unable to solve an unfamiliar science problem (Eylon & Linn, 1988), the subjects were selected from students designated as outstanding in science.

A problem about osmosis was selected because (a) osmosis is a conceptually-complex domain (Friedler, Amir, & Tamir, 1987; Johnstone & Mahmoud, 1980) and (b) high school students are likely to have encountered the topic in a biology course. Thus a problem about osmosis had prospects of generating a meaningful solving course. The problem was selected from Baker & Allen (1967), a college text for general biology. The problem statement was then simplified for this study (see Figure 1).

Insert Figure 1 about here

The problem is also about graphing. The solution is a graph. Graphs, however, are often constructed meaninglessly. The plotting of a graph from data points is generally driven by algorithmic procedural knowledge (Adams & Shrum, 1990; McDermott, Rosenquist, & van Zee, 1987), which could be executed meaninglessly (Landa, 1974). Thus, to obviate a meaningless construction of the graph, the problem was stated qualitatively, that is, the problem statement did not specify numerical values.

A Problem about Osmosis

The figure below is a diagram of an inverted thistle top funnel which can be used to demonstrate osmosis. At the beginning of an experiment there is a dilute solution of sugar and water inside the funnel. An inelastic membrane permeable only to water has been fitted across the immersed funnel opening. The funnel is surrounded by pure water.

Make a graph to show how the solution level in the stem of the funnel changes with time.

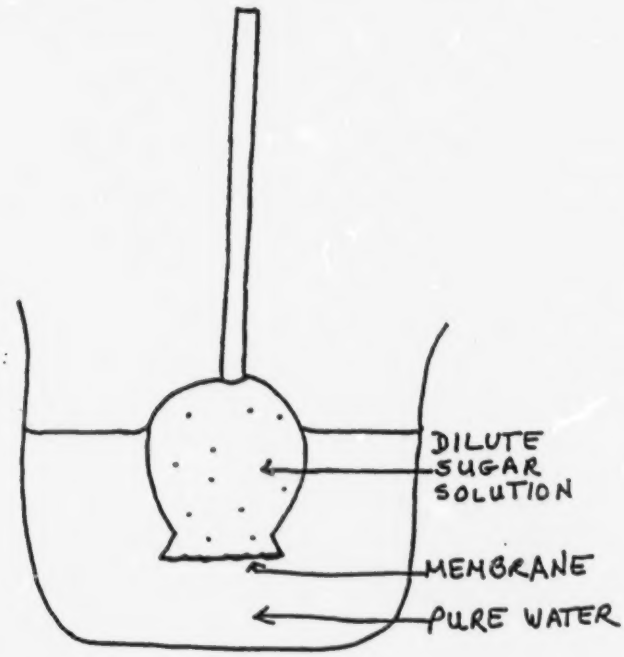

Figure 1. Statement of the problem under study.

Significance

Meaningful problem solving has been a desired outcome of science instruction (McMillan & Swadener, 1991; Stewart, 1983). In this study, an attempt was made to constrain meaningful solving courses by (a) selecting a problem from a conceptually-rich instructional domain, (b) stating the problem qualitatively, and (c) requiring that subjects be unfamiliar with the problem condition. Nevertheless, one solving course was meaningless. An examination of this solving course provokes questions about the interactions between conceptual and procedural knowledge and the effectiveness of strategies designed to secure meaningful problem solving.

Method

Data Collection

The solvers were 16 science students from five suburban public high schools. They had been designated by their science teachers as outstanding both for their conceptual knowledge and interest in problem solving. All had already completed at least two years of high school science, including one year of biology. For each, the investigative course consisted of a presolving clinical interview, followed directly by a think-aloud solving of the problem, and a retrospective report of the solving course. The investigative course

generated verbal data, which were audiorecorded and transcribed, and a pencil-and-paper solution.

The clinical interview. The purpose of the clinical interview was to elicit pieces of conceptual knowledge that could be available for the solving course. The interview tasks were based on a series of diagrams. Some represented osmotic systems. The osmotic systems, however, differed from the problem condition; none had an elevated central tube. Several systems had a U tube (see Figure 2). Intervievees were

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Insert Figure 2 about here

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asked to predict the outcomes of various manipulations of the systems and to justify their predictions.

Other diagrams represented graphs. Intervievees were asked to interpret the meaning of various lines and curves. They were asked to compare various graphs. Moreover, they were asked to construct a graph from a table of data.

The think-aloud protocol. The think-aloud protocol was used to assess the solving course for meaningfulness. The procedure for the think-aloud, including the orientation, was conducted according to the advice and cautions of Ericsson and Simon (1984). The procedure also generated a pencil-and-paper solution, which was assessed for correctness.

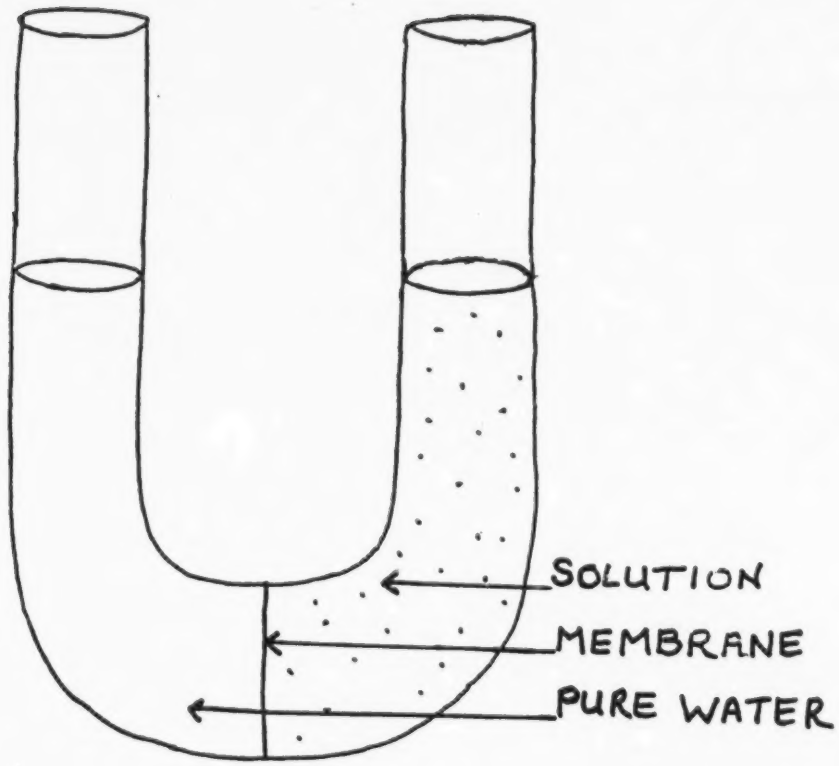

Figure 2. An osmotic U tube.

The retrospective report. The purpose of the retrospective report was to clarify both the think-aloud protocol and the solution and to determine whether the solver had been familiar with the problem condition. Immediately upon finishing the solving course, each solver was asked "How did you solve the problem?" Then the solvers were asked to describe their graph. Subsequently, they were asked whether they had ever before solved a problem like the problem under study. Irrespective of their response, they were directed to describe the problem closest to the one under study that they could ever remember having tried to solve. Finally, they were asked whether, during laboratory experiences, they had ever encountered any conditions like those of the problem.

Data Transformations

Sixteen subjects completed the investigative course. Fourteen of the 16 were judged to have been unfamiliar with the problem condition. They reported no recollection of either having (a) tried to solve a problem wherein a solution rises in an elevated central tube or (b) encountered such a condition during a laboratory experience. Only the data from these 14 were transformed. The data were used to assess whether the solutions were correct and the solving courses were meaningful.

Criteria for correctness. A model solution was constructed in accord with textbook discussions of the problem (see Figure 3). Two attributes of the model

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Insert Figure 3 about here

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solution were then identified as the criteria for correctness:

1. The horizontal axis represented time, and the vertical axis represented the level of the solution.
2. The slope of the graph gradually diminished to zero.

Criteria for meaningfulness and meaninglessness. A solving course was assessed as meaningful if, during the think-aloud protocol or retrospective report, the solver's conceptual knowledge of the problem condition explicitly guided or rationalized procedures to construct the graph. A solving course was assessed as meaningless if such knowledge neither explicitly guided nor rationalized such procedures. Thus, for solvers who categorized the problem as an instance of osmosis, the solving course was assessed as (a) meaningful, if knowledge of the rate of osmosis explicitly guided or rationalized procedures to construct the graph, or (b) meaningless, if such knowledge neither explicitly guided nor rationalized such procedures.

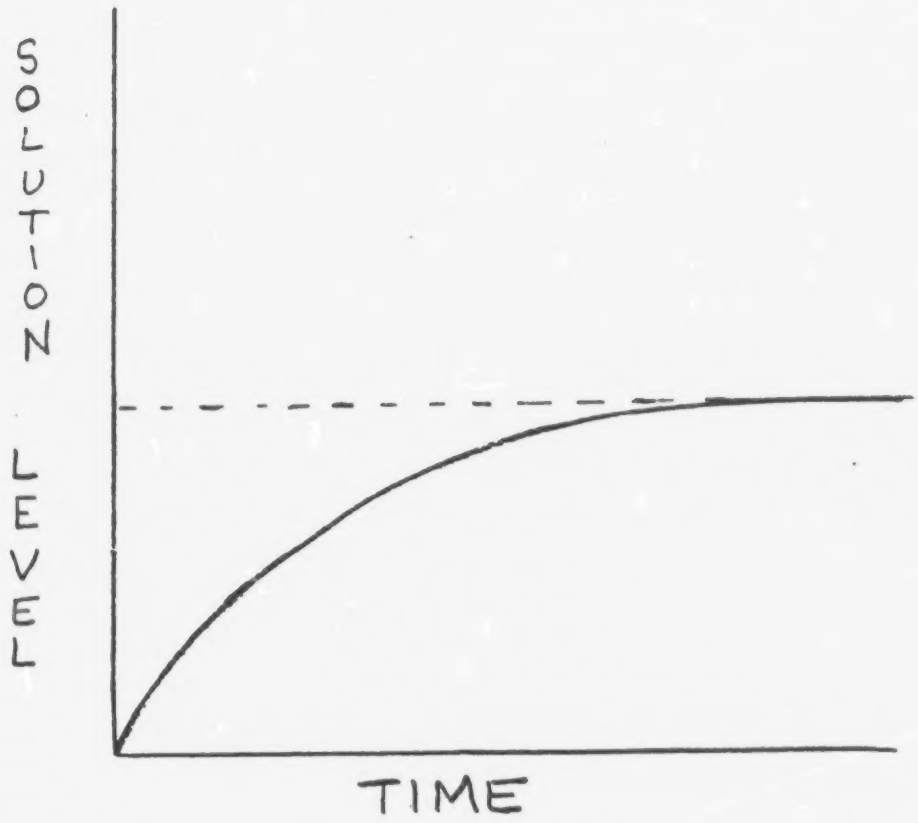

Figure 3. The model solution: Solution level as a function of time.

Results

Thirteen of the 14 solvers found to have been unfamiliar with the problem condition generated a solving course assessed as meaningful. One solving course, however, was assessed as meaningless. It will be argued here that this solving course (a) generated a correct solution, (b) was meaningless, and (c) was an instance of the solving of a problem by analogy.

The Solution

The solving course generated a solution that exhibited the criteria for correctness (see Figure 4).

Insert Figure 4 about here

Moreover, the graph displayed the points that the solver specified in the think-aloud protocol. The solution was, therefore, assessed as correct. Nevertheless, the graph had two flaws. Firstly, the curve was piecewise linear rather than smooth. Secondly, the point at the origin was neither explicit nor connected to the rest of the graph.

The Solving Course

The solving course was assessed as meaningless. The solver's conceptual knowledge of the problem condition neither explicitly guided nor rationalized procedures to construct the graph.

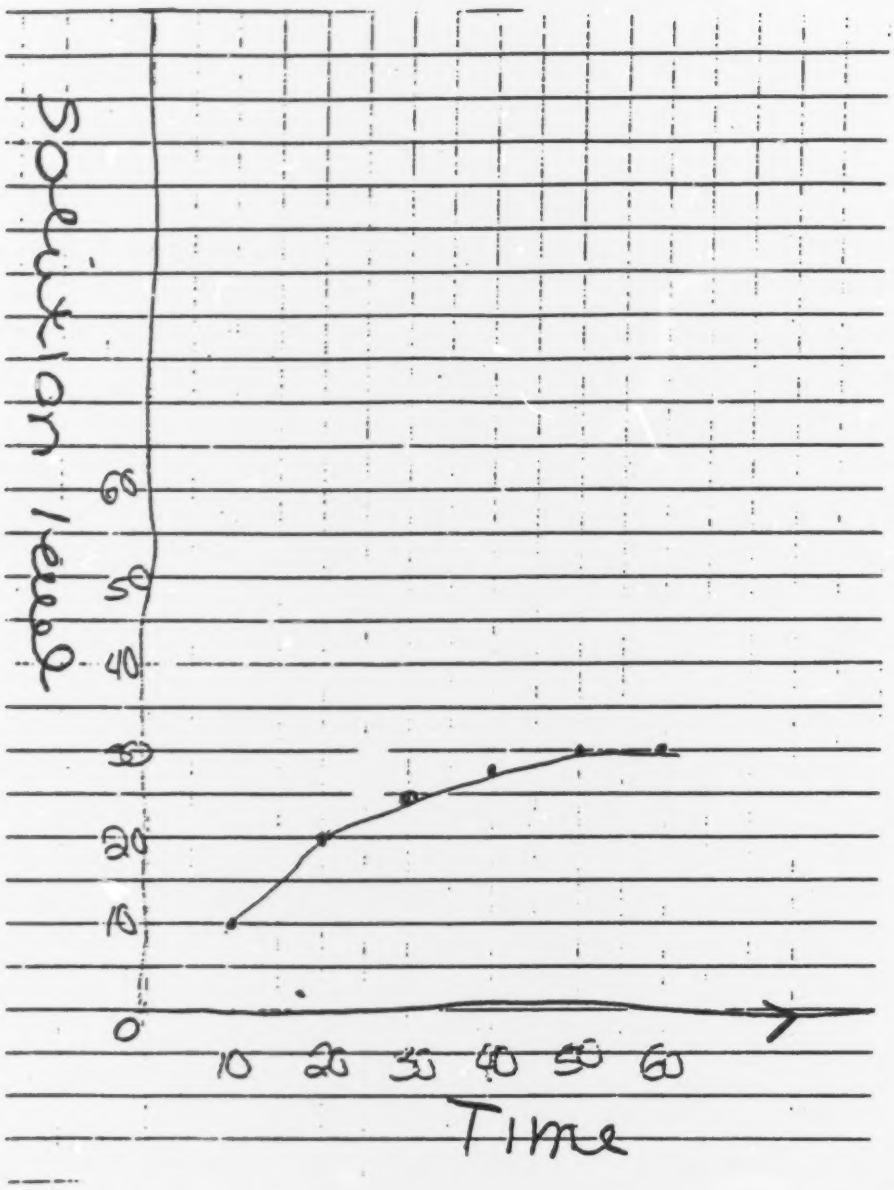

Figure 4. The solution generated by the meaningless solving course.

Data from the think-aloud protocol. The think-aloud protocol consisted of a series of procedures for a point-by-point construction of the graph (see Figure 5):

Insert Figure 5 about here

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1. The solver drew, designated, and labeled the X and Y axes (lines 1-3).

2. She constructed and labeled intervals on the X axis (lines 4-9).

3. On the diagram of the problem condition (see Figure 1), she marked volume gradations along the stem of the funnel (lines 10-12).

4. She constructed and labeled intervals along the Y axis that corresponded to the gradations along the stem of the funnel (lines 13-15).

5. She specified and plotted each of six points (lines 16-31).

6. She inferred that the solution level would remain constant (lines 32-34).

7. She signaled the end of the solving course (line 35).

Knowledge of the rate of osmosis did not seem to guide the specification of the points.

Data from the retrospective report. Yet it was evident that the solver had categorized the problem as

Gonna draw a graph. It's gonna fit here... I'm gonna put time on the Y-- the X axis and then solution level on the Y....

Start at zero. Timewise I'm going to go uhm I'd say in intervals of 10 minutes. Uhm. Each line on the graph is gonna represent 5 minutes. So we're gonna go 5, and then the second line is gonna be 10, then 20, 30, 40... 60. I'm only gonna go up to an hour....

I'm going to mark off measurements on the tube. Each of these measurements is going to represent 5 uhm say milliliters, right? Or something....

Then on the graph, I'm going to label in 10's. The first line is gonna be 5. The second line'll be 10. We're gonna go up in intervals of 5....

Check the solution after 10 minutes, and I'm going to assume maybe that it rises oh I'd say up, maybe up to 10 [plots first point]. I'm gonna check it after 20 minutes, another 10 minutes, and let us say that it rises to maybe, maybe 20, 20 maybe I'd say [plots second point]. And then after 30 minutes, I'm gonna say it rises to 25 [plots third point]; 40 minutes, I'm gonna check, and then it's probably only go to rise to maybe 20-- uh where am I? uhm, after-- wait a minute-- 30 minutes, 30 minutes, 30 minutes was 25. It raised 25. Okay. Forty minutes, it will probably only go to uhm 27 or so [plots fourth point], and then after 50 minutes, it will probably have reached 30 [plots fifth point], and then 60 minutes, it will probably be 30 again [plots sixth point].

And if I checked it after that, it would probably still be 30. It will only have rised 30 milliliters after solution.

I guess that's it....

Figure 5. The think-aloud protocol.

an instance of osmosis. When asked how she solved the problem, the solver referred to the essential attributes of osmosis: a concentration gradient, a semipermeable membrane, and the passage of water across the membrane (Friedler et al., 1987).

> I looked at the solution and since this, well this is a dilute sugar solution uhm, and this is pure water, so the membrane is only permeable to water, and the water is going to go from a higher concentration of water to lower concentration, the dilute sugar solution obviously has a lower concentration of water than pure water does, and pure water is going to diffuse into the membrane, into the solution through the membrane, and the solution, the sugar solution is going to move up the funnel tube.

Moreover, the solver inferred that the water would osmose into rather than out of the funnel. She did not, however, rationalize the points specified on the graph. In fact, her conception of the rate of osmosis ran counter to the knowledge that would have meaningfully generated those points.

Data from the clinical interview. During the clinical interview, the subject appropriately categorized each system as osmotic and consistently predicted that osmosis would proceed at a constant rate. For example, she was given a diagram of a U tube.

The level of the liquid in each arm at both Time t and Time t + X was specified. She was asked to mark the level in each arm at Time t + 2X. Her markings and justification indicate a conception that osmosis proceeds at a constant rate (see Figure 6).

Insert Figure 6 about here

In response to the diagrams of graphs, she consistently associated a constant rate with a straight line rather than with either a smooth or piecewise linear curve. For example, she interpreted a line with a negative slope by contrasting it to a curve with a negative slope that gradually increased to zero: "It's, it's more of a steady rate. I d-- I don't really know the word that I want for that, but it's more of a steady rate than this [curved] one."

For another task, she constructed a piecewise linear curve in response to a table of data and interpreted that graph as representing a rate that was "not steady." When asked how she could tell from the graph that the rate was not steady, she answered: "You can see that it's really not occurring in a straight uh uh uhm the same rate because if it was, it would be just a straight line." Thus she appropriately interpreted straight lines as representing steady rates

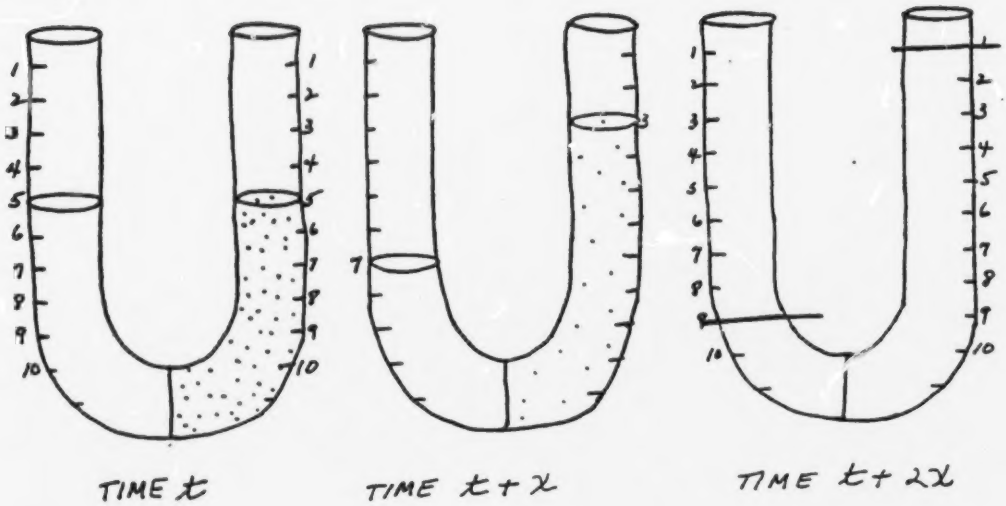

Figure 6. A clinical interview task that elicited the subject's conception of the rate of osmosis.

and a piecewise linear curve as representing a rate that was not steady.

Yet, despite her conception that osmosis proceeds at a constant rate, she constructed a piecewise linear curve to solve the problem. Thus the assessment of meaninglessness is supported by the fact that her conception of the rate of osmosis ran counter to the knowledge that would have meaningfully generated her graph.

Solving by Analogy

It is argued that this meaningless solving course was analogical rather than algorithmic. It seems to have been driven by a similar problem rather than by a prescribed routine.

The points of the graph seem to have been generated from the recollection of a particular laboratory event, which, like the problem under study, the solver had also categorized as an instance of osmosis. A later portion of the retrospective report supports this argument:

I: Now, have you ever solved a problem like this before?

S: Uhm. Kind of. Sort of.

I: What's the closest problem you can remember?

S: Uhm. I think a lab last year in chemistry.

I: Was it set up like this?

S: Uhm. It didn't have a funnel tube. I think it was more the U tube. It didn't have the funnel. It was a U.... It was similar in that we used a sugar solution. Uhm, but it was different in that we, we used a very concentrated sugar solution to see how much the water would actually go into-- how, how she exaggerated the sugar solution so that we could see that the act-- the water would actually diffuse through the membrane....

I: Did you recognize the problem in the lab when, when I showed you this problem?

S: Uhm. Oh, well, I-- yeah, I recognized that it was a, a similar problem. Uhm. I know it's not exactly the same, but it's similar, pretty much similar.

In fact, her categorization of the problem may have been the cue to reconstruct this laboratory event as the analogue. Her marking volume graduations along the stem of the funnel suggests that she had confirmed the correspondence and was preparing to use the analogue to

generate the points of the graph. Thus the protocol and retrospective report exemplified an analogical solving process (Clement, 1981; Gick & Holyoak, 1980).

The final portion of the retrospective report substantiates the claim that the laboratory event was the analogue. The solver was asked, "When you did the lab, what kinds of things were you looking for?"

> The amount of time that she alloved us, and each time after it-- we didn't do it in an hour. Actually, we might have. It was a double period. So uhm she made us check it like every ten or five minutes.

Thus the intervals on the X axis (see Figure 4) were congruent with her recollection of the laboratory event.

Discussion

For a graphing problem about osmosis, this paper documents both (a) a correct solution generated by a meaningless solving course for an unfamiliar problem and (b) the meaningless solving of a problem by analogy. Thus a correct solution to an unfamiliar problem need not indicate that the solving course was meaningful. Moreover, a procedure other than a prescribed algorithm can be executed meaninglessly. These findings are an addition to the science education literature.

These findings are tightly linked to the import of correct, unfamiliar, and meaningful/meaningless. The solution was assessed as correct despite its flaws. McDermott et al. (1987) have reported that the joining of points in a piecewise linear rather than a smooth curve is a prevalent error even among advanced students. Thus this error and the failure to indicate the origin of the graph were not deemed sufficient to assess the solution as incorrect.

It has been argued that the solver was unfamiliar with the problem. Although she was certainly familiar with osmotic systems, she had no recollection of a problem or laboratory system with the particular problem condition. Instead, she recalled a similar system that proved useful.

To access any procedure, even one executed meaninglessly, the solver must first categorize the problem (Chi, Feltovich, & Glaser, 1981). Thus no solving course can be absolutely independent of the solver's conceptual knowledge (Stevart, 1982a). The essence of the problem under study, however, was to represent the rate at which the solution level rose. Although operations were accessed and rationalized with conceptual knowledge of osmosis, the specification of the points of the graph was independent of the solver's knowledge of the rate of osmosis. Thus the solving course was assessed as meaningless.

The solver recalled the results of a laboratory event but apparently had never constructed their meaning. She did not know that osmosis proceeds at a diminishing rate. When asked what kinds of things she was looking for during the laboratory experience, she mentioned procedures rather than conceptions. Thus, not only was the solving course meaningless, but the laboratory experience was probably meaningless as well.

In fact, laboratory experiences are often meaningless (Champagne, Gunstone, & Klopfer, 1985). Students follow directions and record data without thinking about their import (Tamir & Lunetta, 1981). The findings from this study suggest that meaningless laboratory experiences can generate other meaningless cognitive activities.

Implications for Research

The findings from this study also provoke questions concerning the interaction between conceptual and procedural knowledge during problem solving. What determines whether conceptual or procedural knowledge will be used at any given point? What induces a solver to shift between conceptual and procedural knowledge? Is there a threshold of conceptual knowledge that is necessary for the meaningful solving of a given problem?

Implications for Instruction

For this study, a correct solution to an unfamiliar problem was generated by a solver whose conceptual knowledge ran counter to public science knowledge. Thus these findings substantiate (a) the importance of attending to the solving course as well as the answer to even an unfamiliar problem and (b) the need for students to solve and/or discuss their solving courses with others.

The findings from this study also substantiate the need to make laboratory experiences more meaningful. Various researchers have recommended Vee diagramming (Lehman, Carter, & Kahle, 1985; Novak & Gowin, 1984). Moreover, suggestions that have been made to promote meaningful problem solving can be extended to laboratory experiences as well.

The problem solving techniques suggested by Driver (1983) and Reif (1983) to help novices access relevant principles can also be used to focus attention on the meaning of a laboratory experience. Driver has recommended guiding students with specific questions in order to teach them what to see. Furthermore, she has suggested giving individuals and groups the opportunity to talk about their interpretations. Similarly, Reif has recommended having students "redescribe" a problem in terms of the relevant concepts. Laboratory events can be so guided and redescribed as well. In fact, the

small groups typical of most laboratory activities provide an interactive context that can be used to promote more meaningful laboratory activities (Lunetta, 1990).

References

Adams, D. D., & Shrum, J. W. (1990). The effects of microcomputer-based laboratory exercises on the acquisition of line graph construction and interpretation skills by high school biology students. Journal of Research in Science Teaching, 27(8), 777-787.

Anderson, J. R. (1983). The architecture of cognition. Cambridge, MA: Harvard University Press.

Baker, J. J. W., & Allen, G. E. (1967). The study of biology. Reading, MA: Addison-Wesley.

Champagne, A. B., Gunstone, R. F., & Klopfer, L. E. (1985). Instructional consequences of students' knowledge about physical phenomena. In L. H. T. West & A. L. Pines (Eds.), Cognitive structure and conceptual change (pp. 61-90). New York: Academic Press.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. Cognitive Science, 5, 121-152.

Clement, J. (1981). Analogy generation in scientific problem solving. Amherst, MA: University of Massachusetts, Department of Physics and Astronomy. (ERIC Document Reproduction Service No. ED 228 044)

Driver, R. (1983). The pupil as scientist? Milton Keynes: Open University Press.

Ericsson, K. A., & Simon, H. A. (1984). Protocol analysis. Cambridge, MA: MIT Press.

Eylon, B., & Linn, M. C. (1988). Research perspectives in science education. Review of Educational Research, 58(3), 251-301.

Friedler, Y., Amir, R., & Tamir, P. (1987). High school students' difficulties in understanding osmosis. International Journal of Science Education, 9(5), 541-551.

Gabel, D. L., Sherwood, R. D., & Enochs L. (1984). Problem-solving skills of high school chemistry students. Journal of Research in Science Teaching, 21(2), 221-233.

Gagne, R. M. (1968, November). Learning hierarchies [Presidential Address of Division 15 on August 31, 1968, San Francisco, CA]. Educational Psychologist, 6(1), 1-9.

Gick, M. L., & Holyoak, K. (1980). Analogical problem solving. Cognitive Psychology, 12, 306-355.

Greeno, J. G. (1973). The structure of memory and the process of solving problems. In R. L. Solso (Ed.), Contemporary issues in cognitive psychology: The Loyola Symposium (pp. 104-133). Washington, D. C.: V. H. Winston.

Greeno, J. G. (1978). Understanding and procedural knowledge in mathematics instruction. Educational Psychologist, 12, 262-283.

Greeno, J. G. (1980). Trends in theory of knowledge for problem solving. In D. T. Tuma & F. Reif (Eds.), Problem solving and education: Issues in teaching and research (pp. 9-23). Hillsdale, NJ: L. Erlbaum Associates.

Johnstone, A. H., & Mahmoud, N. A. (1980). Isolating topics of high perceived difficulty in school biology. Journal of Biological Education, 14(2), 163-166.

Landa, L. N. (1974). Algorithmization in learning and instruction (F. F. Kopstein, Ed. & V. Bennett, Trans.). Englewood Cliffs, NJ: Educational Technology Publications.

Larkin, J. H. (1980). Teaching problem solving in physics: Psychological laboratory and the practical classroom. In D. T. Tuma & F. Reif (Eds.), Problem solving and education: Issues in teaching and research (pp. 111-125). Hillsdale, NJ: L. Erlbaum Associates.

Lehman, J. D., Carter, C., & Kahle, J. B. (1985). Concept mapping, Vee mapping, and achievement: Results of a field study with black high school students. Journal of Research in Science Teaching, 22(7), 663-673.

Lunetta, V. N. (1990). Cooperative learning in science, mathematics, & computer problem solving. In M. Gardner, J. G. Greeno, F. Reif, A. H. Schoenfeld, A. diSessa, & E. Stage (Eds.), Toward a scientific practice of science education (pp. 235-249). Hillsdale, NJ: L. Erlbaum Associates.

Lythcott, J. (1990). Problem solving and requisite knowledge of chemistry. Journal of Chemical Education, 67(3), 248-252.

McDermott, L. C., Rosenquist, M. L., & van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. American Journal of Physics, 55(6), 503-513.

McMillan, C., III, & Swadener, M. (1991). Novice use of qualitative versus quantitative problem solving in electrostatics, Journal of Research in Science Teaching, 28(8), 661-670.

Novak, J. D. (1977). An alternative to Piagetian psychology for science and mathematics education. Science Education, 61(4), 453-477.

Novak, J. D., & Gowin, D. B. (1984). Learning how to learn. New York: Cambridge University Press.

Novak, J. D., Gowin, D. B., & Johansen, G. T. (1983). The use of concept mapping and knowledge vee mapping with junior high school science students. Science Education, 67(5), 625-645.

Polya, G. (1957). How to solve it: A new aspect of mathematical method (2nd ed.). Princeton, NJ: Princeton University.

Reif, F. (1983). How can chemists teach problem solving? Journal of Chemical Education, 60(11), 948-953.

Ryle, G. (1949). The concept of mind. London: Hutchinson.

Shuell, T. J. (1985). Knowledge representation, cognitive structure, and school learning: A historical perspective. In L. H. T. West & A. L. Pines (Eds.), Cognitive structure and conceptual change (pp. 117-130). New York: Academic Press.

Siegler, R. S. (1986). Children's thinking. Englewood Cliffs, NJ: Prentice-Hall.

Stewart, J. H. (1982a). Difficulties experienced by high school students when learning basic Mendelian genetics. American Biology Teacher, 44(2), 80-89.

Stewart, J. (1982b). Two aspects of meaningful problem solving in science. Science Education, 66(5), 731-749.

Stewart, J. (1983). Student problem solving in high school genetics. Science Education, 67(4), 523-540.

Tamir, P., & Lunetta, V. N. (1981). Inquiry related tasks in high school science laboratory handbooks. Science Education, 65(5), 477-484.

Wertheimer, M. (1959). Productive thinking (enlarged ed.). Michael Wertheimer (Ed.). New York: Harper and Row. (Original work published 1945)

Yarroch, W. L. (1985). Student understanding of chemical equation balancing. Journal of Research in Science Teaching, 22(5), 449-459.